Zadaci 6

1. zadatak

(%i1) eq1: 'diff(y, t) + y = cos(t);

(%o1) $\dfrac{d}{d\,t}\,y + y = \cos(t)$

(%i2) s1: ode2(eq1, y, t);

(%o2) $y = \%e^{-t}\left(\dfrac{\%e^{t}\left(\sin(t)+\cos(t)\right)}{2} + \%c\right)$

(%i4) s1: expand(s1);

(%o4) $y = \dfrac{\sin(t)}{2} + \dfrac{\cos(t)}{2} + \%c\,\%e^{-t}$

(%i5) s1: rhs(s1);

(%o5) $\dfrac{\sin(t)}{2} + \dfrac{\cos(t)}{2} + \%c\,\%e^{-t}$

(%i6) eqic1: ev(s1, t = 0);

(%o6) $\%c + \dfrac{1}{2}$

(%i8) solve(eqic1 = 1/2, %c);

(%o8) $[\,\%c = 0\,]$

(%i9) sy: subst(0, %c, s1);

(%o9) $\dfrac{\sin(t)}{2} + \dfrac{\cos(t)}{2}$

provera

(%i10) p1: subst(sy, y, eq1);

(%o10) $\dfrac{d}{d\,t}\left(\dfrac{\sin(t)}{2} + \dfrac{\cos(t)}{2}\right) + \dfrac{\sin(t)}{2} + \dfrac{\cos(t)}{2} = \cos(t)$

(%i11) p1: ev(p1, diff);

(%o11) $\cos(t) = \cos(t)$

(%i12) is(p1);

(%o12) true

2. zadatak

(%i13) eq2: 'diff(y, t) - 2 * y = - t^2;

(%o13) $\dfrac{d}{d\,t}\,y - 2\,y = -\,t^2$

(%i14) s2: ode2(eq2, y, t);

(%o14) $y = \left(\dfrac{\left(2\,t^2 + 2\,t + 1\right)\,\%e^{-\,2\,t}}{4} + \%c\right)\%e^{2\,t}$

(%i15) s2: expand(s2);

(%o15) $y = \%c\,\%e^{2\,t} + \dfrac{t^2}{2} + \dfrac{t}{2} + \dfrac{1}{4}$

(%i16) s2: rhs(s2);

(%o16) $\%c\,\%e^{2\,t} + \dfrac{t^2}{2} + \dfrac{t}{2} + \dfrac{1}{4}$

(%i17) eqic2: ev(s2, t = 0);

(%o17) $\%c + \dfrac{1}{4}$

(%i18) solve(eqic2 = 1/4, %c);

(%o18) $[\%c = 0]$

(%i19) sy: subst(0, %c, s2);

(%o19) $\dfrac{t^2}{2} + \dfrac{t}{2} + \dfrac{1}{4}$

provera

(%i20) p2: subst(sy, y, eq2);

$$(\%o20) \quad \frac{d}{dt}\left(\frac{t^2}{2}+\frac{t}{2}+\frac{1}{4}\right) - 2\left(\frac{t^2}{2}+\frac{t}{2}+\frac{1}{4}\right) = -t^2$$

(%i21) p2: ev(p2, diff);

$$(\%o21) \quad -2\left(\frac{t^2}{2}+\frac{t}{2}+\frac{1}{4}\right) + t + \frac{1}{2} = -t^2$$

(%i22) p2: ratsimp(p2);

$$(\%o22) \quad -t^2 = -t^2$$

(%i23) is(p2);

(%o23) true

3. zadatak

(%i24) eq3: 'diff(y, t) + 2*y = 2*t;

$$(\%o24) \quad \frac{d}{dt}y + 2y = 2t$$

(%i25) s3: ode2(eq3, y, t);

$$(\%o25) \quad y = \%e^{-2t}\left(\frac{(2t-1)\,\%e^{2t}}{2} + \%c\right)$$

(%i26) s3: expand(s3);

$$(\%o26) \quad y = \%c\,\%e^{-2t} + t - \frac{1}{2}$$

(%i27) s3: rhs(s3);

$$(\%o27) \quad \%c\,\%e^{-2t} + t - \frac{1}{2}$$

(%i28) eqic3: ev(s3, t = 0);

$$(\%o28) \quad \%c - \frac{1}{2}$$

(%i29) solve(eqic3 = -1, %c);

(%o29) $[\%c = -\frac{1}{2}]$

(%i30) sy: subst(-1/2, %c, s3);

(%o30) $-\frac{\%e^{-2t}}{2} + t - \frac{1}{2}$

provera

(%i31) p3: subst(sy, y, eq3);

(%o31) $\frac{d}{dt}\left(-\frac{\%e^{-2t}}{2} + t - \frac{1}{2}\right) + 2\left(-\frac{\%e^{-2t}}{2} + t - \frac{1}{2}\right) = 2t$

(%i32) p3: ev(p3, diff);

(%o32) $\%e^{-2t} + 2\left(-\frac{\%e^{-2t}}{2} + t - \frac{1}{2}\right) + 1 = 2t$

(%i33) p3: ratsimp(p3);

(%o33) $2t = 2t$

(%i34) is(p3);

(%o34) true